Modeling Monetary Policy

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Outline of the presentation

- some motivations
- intuition of what we do
- model
- results
- conclusions

In models used for monetary policy analysis, relationship between policy instrument & target variables given by consumption Euler equation:

$$\frac{1}{1+r_t} = \beta E_t \left(\frac{u_{c,t+1}}{u_{c,t}} \pi_{t+1}^{-1} \right)$$

$$r_t = -E_t \log \left(\frac{c_t}{c_{t+1}} \right) + E_t \log \pi_{t+1} + \text{constant}$$

Issue:

Empirically, the policy instrument is not related to the policy “targets”, i.e. inflation and consumption, in the way implied by the Euler consumption equation.

Computed Euler rate

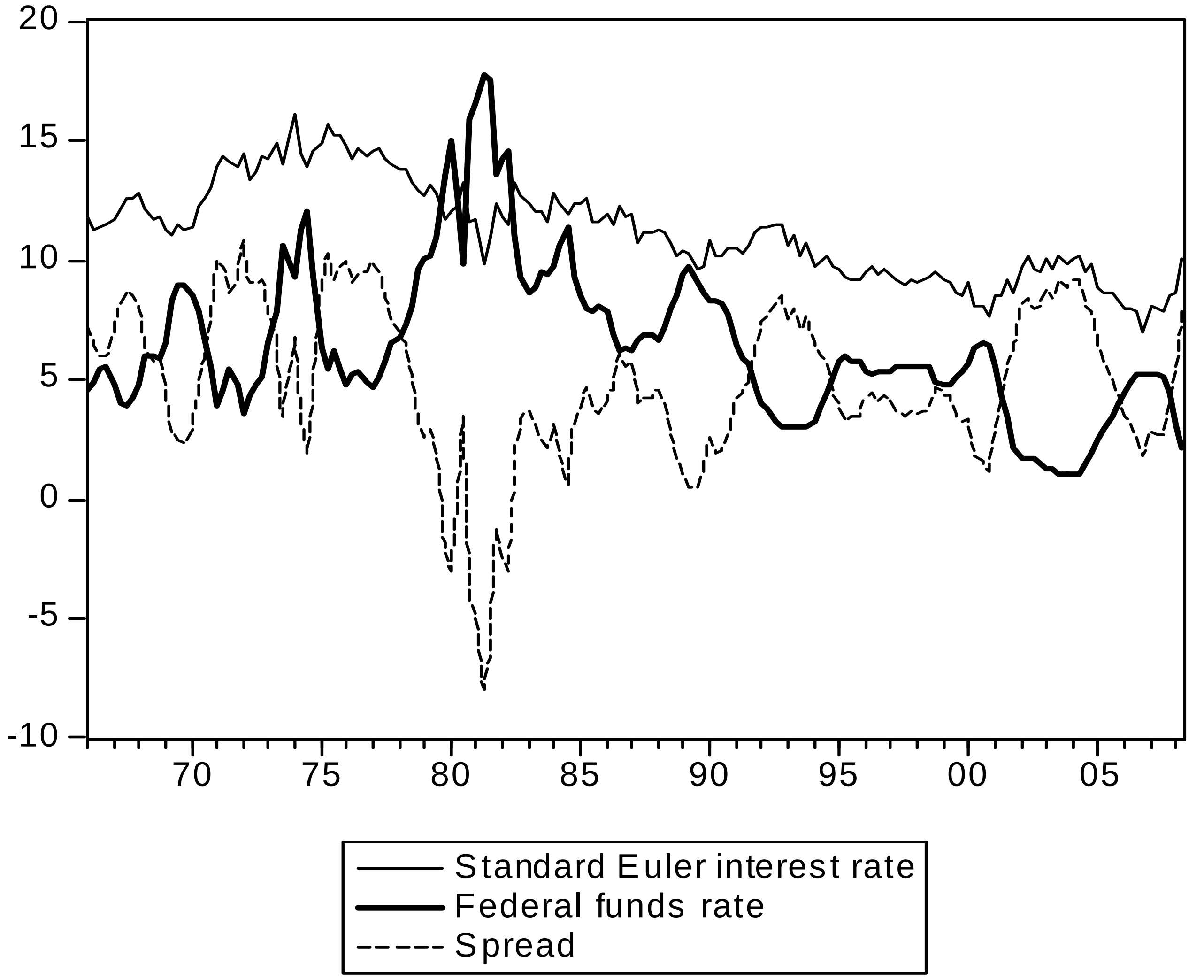
- Interest rate implied by Euler equations (Canzoneri et al., 2007)
  - Rate computed with a standard consumption Euler equation (CRRA preferences)

$$\frac{1}{1+r_t^d} = \beta E_t \left(\frac{u_{c,t+1}}{u_{c,t}} \frac{P_t}{P_{t+1}} \right)$$

  - Using conditional moments from a six-variable VAR (Fuhrer, 2000)

- Comparison of the computed standard Euler rate r^d, the fed funds rate r,
  - and the spread between these two rates, $r_t^d - r_t$.

Euler rate and federal funds rates (in %, quarterly data)



HP-filtered Euler rate and (negative) federal funds rate

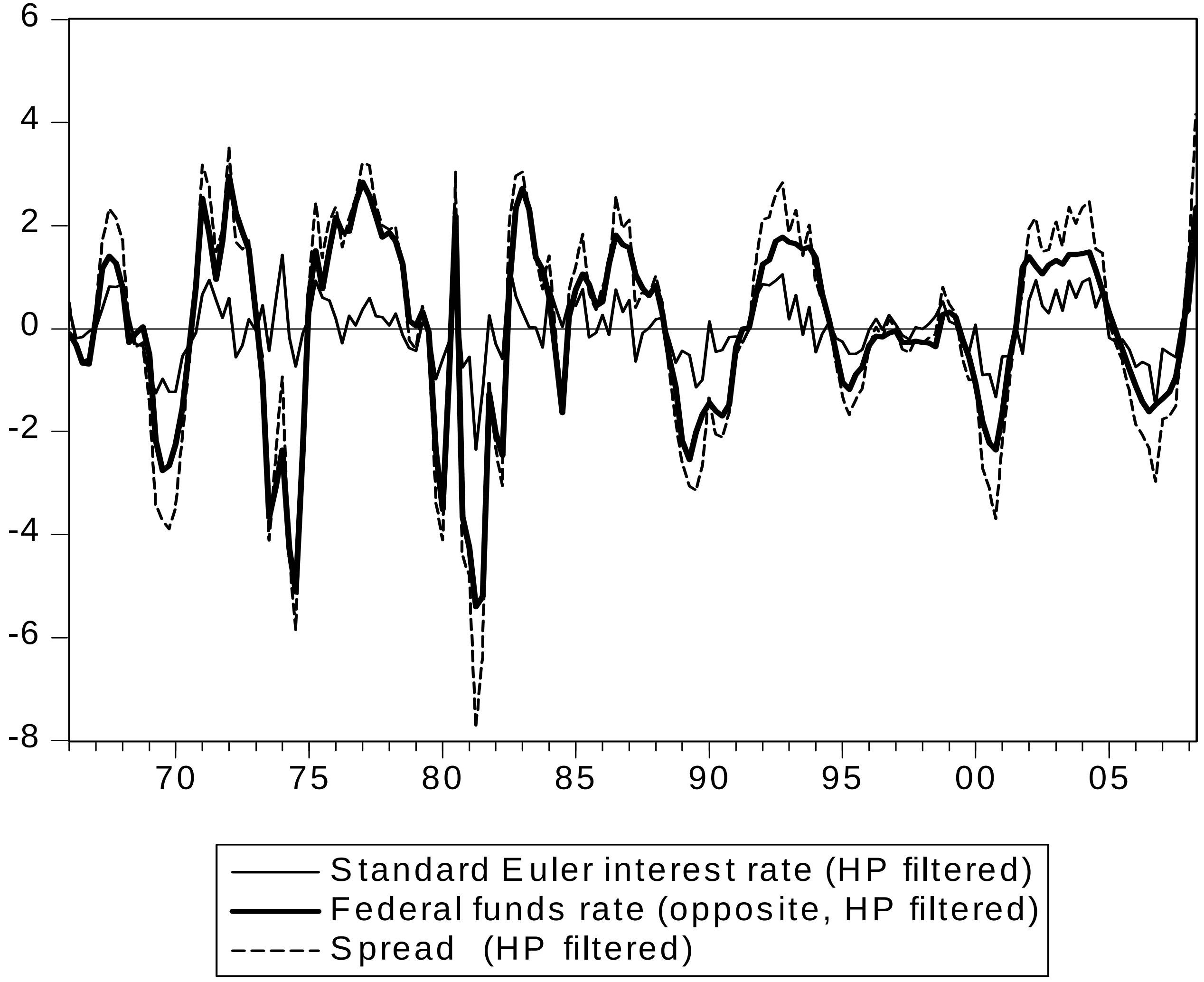


Table 1 Empirical correlations

	Standard Euler equation	Our model's Euler equation
HP-filtered rates		
$\text{corr}(\widetilde{s_1}, \widetilde{r})$	-0.98	-0.90
$\text{corr}(\widetilde{r^d}, \widetilde{r})$	-0.66	-0.57

- spread between Euler rate and policy rate :

$$\left[-E_t \log\left(\frac{c_t}{c_{t+1}}\right) + E_t \log \pi_{t+1} + \text{constant}\right] - r_t^m$$

- highly negative correlation between policy rate and spread as policy instrument moves much more than inflation and consumption growth
  - in theory, spread is zero, i.e. one-for-one movements
  - thus standard models capture essentially none of the link between policy and the macroeconomy
- negative correlation between policy rate and Euler rate as policy real interest rate and consumption growth are negatively correlated empirically
  - thus the two rates (Euler and policy) than theory equates are negatively correlated empirically

In contrast,

- we model central banks in what they actually do, i.e. influencing the return on assets traded in open market operations
  - thus short-term interest rate movements related to liquidity premium movements, different from Euler rate
  - average positive spread between the Euler and policy rates, and negative correlation between policy rate and spread
- transmission mechanism: CB affects value and evolution of “collateral” used in OMO
  - increase in interest rate will be associated with decreasing consumption growth, as “collateral” decreases; thus negative correlation between policy and Euler rates

In our setup, the central bank (CB) injects money not in a lump-sum way, but by discounting eligible assets in open market operations (OMO), via repo & outright purchases/sales:

OMO (simplified): **M=B/Rm**, where M is money, B is eligible asset (government bonds), and Rm is the policy (discount) rate.

The endogenous rate R on eligible bonds will approximately equal E(Rm), thus CB can bring R down lower than the otherwise (if not accepted in OMO) market rate. This liquidity premium will be negatively correlated with policy rate.

CB transfers only interest earnings back to households, not the change in money stock; thus wealth is held at CB as counterpart for outstanding money. This will generate hump-shape consumption responses to shocks.

Timing of events

(detailed derivations in paper)

- Beginning of the period t:

A household (HH) i enters a period with assets carried over from the previous period

$$M^H_{i,t-1} + B_{i,t-1} + D_{i,t-1}$$

where M^H denotes money, B gov. bonds, and D privately issued debt.

1. Aggregate shocks materialize, labor is supplied, and goods are produced by firms.
2. HH enters the money market, where money can be traded only in exchange for eligible assets. CB only accepts government bonds in exchange for money. The repo rate R^m_t is controlled by the CB. Open market constraint:

η_t

$$I_{i,t} \leq B_{i,t-1}/R^m_t$$

where $I_{i,t}$ is new money.

3. HH enters the goods market, where it needs money to buy goods.

   Goods market constraint:

$$P_t c_{i,t} \leq I_{i,t} + M^H_{i,t-1}$$

4. Income and dividends δ_t are paid back in cash, and bonds can be repurchased.

   In the asset market, new bonds are issued and HH trade bonds, debt and money. The asset market constraint can be written as

λ_t

$$\begin{aligned}&(B_{i,t}/R_t) + E_t[q_{t,t+1} D_{i,t}] + M^H_{i,t} + (R^m_t - 1)\, I_{i,t} \\ &\leq B_{i,t-1} + D_{i,t-1} + M^H_{i,t-1} + P_t\,(w_t n_{i,t} + \delta_{i,t} - c_{i,t}) + P_t \tau_t,\end{aligned}$$

   where $P_t \tau_t$ are lump-sum transfers (of bonds), $q_{t,t+1}$ is the stochastic discount factor, and R_t is the bond rate.

   CB reinvests to the amount that equals its stock of maturing bonds and does not change money supply.

$q_{t,t+1}$ is the stochastic discount factor.

debt rate $R^d_t = [E_t q_{t,t+1}]^{-1}$

- End of the period

Private sector I/III

- Continuum of infinitely lived households
  - Household $i \in [0,1]$ supplies labor n, consumes c, and aims at maximizing

$$E \sum_{t=0}^{\infty} \beta^t u\left(c_{i,t}, n_{i,t}\right), \quad \beta \in (0,1),$$

  - invests in money M^H (M^R), bonds B, nominally state contingent claims D,

$$\frac{B_{i,t}}{R_t} + E_t[q_{t,t+1} D_{i,t}] + M_{i,t}^H + (R_t^m - 1)\, I_{i,t} \tag{3}$$
$$= B_{i,t-1} + D_{i,t-1} + M_{i,t-1}^H + P_t w_t n_{i,t} - P_t c_{i,t} + P_t \delta_{i,t} + P_t \tau_t$$

where $P_t \delta_{i,t}$ are profits and $q_{t,t+1}$ is the stochastic discount factor.

Private sector II/III

- Maximizing welfare s.t. money, goods, and asset market constraints (1), (2), (3)

$$
\begin{aligned}
\beta E_t[u_c(c_{i,t+1})/\pi_{t+1}] &= -u_n(n_{i,t})/w_t \\
\frac{E_t[(1/R^m_{t+1}) \cdot u_c(c_{i,t+1})\pi^{-1}_{t+1}]}{E_t[u_c(c_{i,t+1})\pi^{-1}_{t+1}]} &= 1/R_t \\
\beta \frac{E_t[(\lambda_{t+1} + \eta_{t+1})\pi^{-1}_{t+1}]}{\lambda_t} &= 1/R_t \\
\beta \frac{E_t[\lambda_{t+1}\pi^{-1}_{t+1}]}{\lambda_t} &= 1/R^d_t
\end{aligned}
$$

where η_t measures the liquidity value of bonds.

- The debt rate $R^d_t = [E_t q_{t,t+1}]^{-1}$ corresponds to the consumption Euler rate.

Private sector III/III

- Competitive firms
  - produce final goods using differentiated goods $y_t = \int_0^1 y_{jt} dj$ and sell it at P_t
- Monopolistically competitive firms
  - produce differentiated goods $y_{j,t}$ with $j \in [0, 1]$ under monopolistic competition

$$y_{j,t} = a_t n_{j,t}^{\alpha}$$

where $n_j = \int n_i^j di$ and a_t is an economy-wide stochastic productivity level

  - Max. profits s.t. $y_{i,t} = (P_{it}/P_t)^{-\epsilon} y_t$ and non-zero probability of fixed prices

$$\begin{aligned} w_t &= mc_{j,t} \alpha y_{j,t}/n_{j,t} \\ \widehat{\pi}_t &= \kappa \widehat{mc}_{j,t} + \beta E_t \widehat{\pi}_{t+1} \end{aligned}$$

where $\widehat{mc}_{j,t}$ denotes marginal costs and $\widehat{x}_t$ log deviations from a steady state.

Public sector I/II

- Central bank
  - Instrument(s): CB sets the repo rate R_t^m

$$R_t^m = R^m \left(R_{t-1}^m, \pi_t, \varepsilon_t \right)$$

    and controls the ratio between repos and outright sales: $M_t^R = \Omega \cdot M_t^H$

  - CB transfers earnings from asset holdings to the fiscal authority:

$$P_t \tau_t^m = B_t^c \left(1 - 1/R_t \right)$$

  - Its bond holdings will evolve according to

$$B_t^c - B_{t-1}^c = R_t^m I_t - \left[I_t - \left(M_t^H - M_{t-1}^H \right) \right]$$

Public sector II/II

- Fiscal authority
  - Receives CB transfer and issues one-period bonds B_t^T at the price $1/R_t$

$$\left(B_t^T/R_t\right) + P_t\tau_t^m = B_{t-1}^T + P_t\tau_t$$

where $P_t\tau_t$ denotes lump-sum transfers.

  - Short-term bonds are supplied at a constant rate

$$B_t^T = \Gamma B_{t-1}^T$$

where the growth rate satisfies $\Gamma > \beta$.

Equilibrium I/II

- No arbitrage opportunities and markets clear, e.g. $\int D_{i,t}di = 0$,

$$I_t = M_t^H - M_{t-1}^H + M_t^R, \qquad B_t^T = B_t + B_t^c,$$

where $\int B_{i,t}di = B_t$, $\int M_{i,t}^H di = M_t^H$, and $\int M_{i,t}^R di = M_t^R$

- Stock of government bonds held by the private sector
  - decreases with the amount and the relative price of new money $R_t^m I_t$

$$B_t - B_{t-1} = (\Gamma - 1)\, B_{t-1}^T - R_t^m I_t + M_t^R$$

and increases with the share of money held under repos.

Equilibrium II/II

- Open market constraint will be binding if $\eta_t = (u_{i,nt}/w_t) + (u_{i,ct}/R_t^m) > 0$
  - Corresponds to the difference between the price of bonds and of debt

$$\beta E_t \left[\eta_{t+1}\pi_{t+1}^{-1}\right]/\lambda_t = (1/R_t) - \left(1/R_t^d\right)$$

  - Money supply then depends on the discounted value of bonds

$$B_{t-1}/R_t^m = M_t^H - M_{t-1}^H + M_t^R$$

- CB sets the long-run repo rate at a sufficiently low level $R^m \in (1, \pi/\beta)$
  - Local analysis in the neighborhood of a steady state with $\eta > 0$
  - Comparison with the standard case $R^m = \pi/\beta$ such that $\eta = 0$

Bond rate vs. debt rate I/II

- Relates to the (empirical) comparison: federal funds rate vs. Euler rate

- Simplified version: flexible prices, only repo money, log-utility, exog. repo rate

Proposition 1 *The spread between the debt rate R_t^d and the bond rate R_t decreases with the current level and the variance of the repo rate.*

- Higher repo rate reduces the amount of cash per unit of bonds
  - Valuation of bonds and the liquidity premium decrease
  - Similar effect when the repo rate is more volatile

Bond rate vs. debt rate II/II

- Numerical results for a version with sticky prices, money supplied with $\Omega = 0.5$ and a steady state spread equal to 120 b.p.

Table 3 Unconditional correlations

	Interest rate shocks		Technology shocks
	$\rho_\pi = 0$	$\rho_\pi = 1.5$	$\rho_\pi = 1.5$
corr(s_1, R^m)	-0.996	-0.997	-0.861
corr$\left(R^d, R^m\right)$	-0.885	-0.907	-0.830

- Compared with the empirical correlations
  - Correlation for the spread under technology shocks is consistent
  - Correlations for the debt rate is too high

Repo rate vs. bond rate

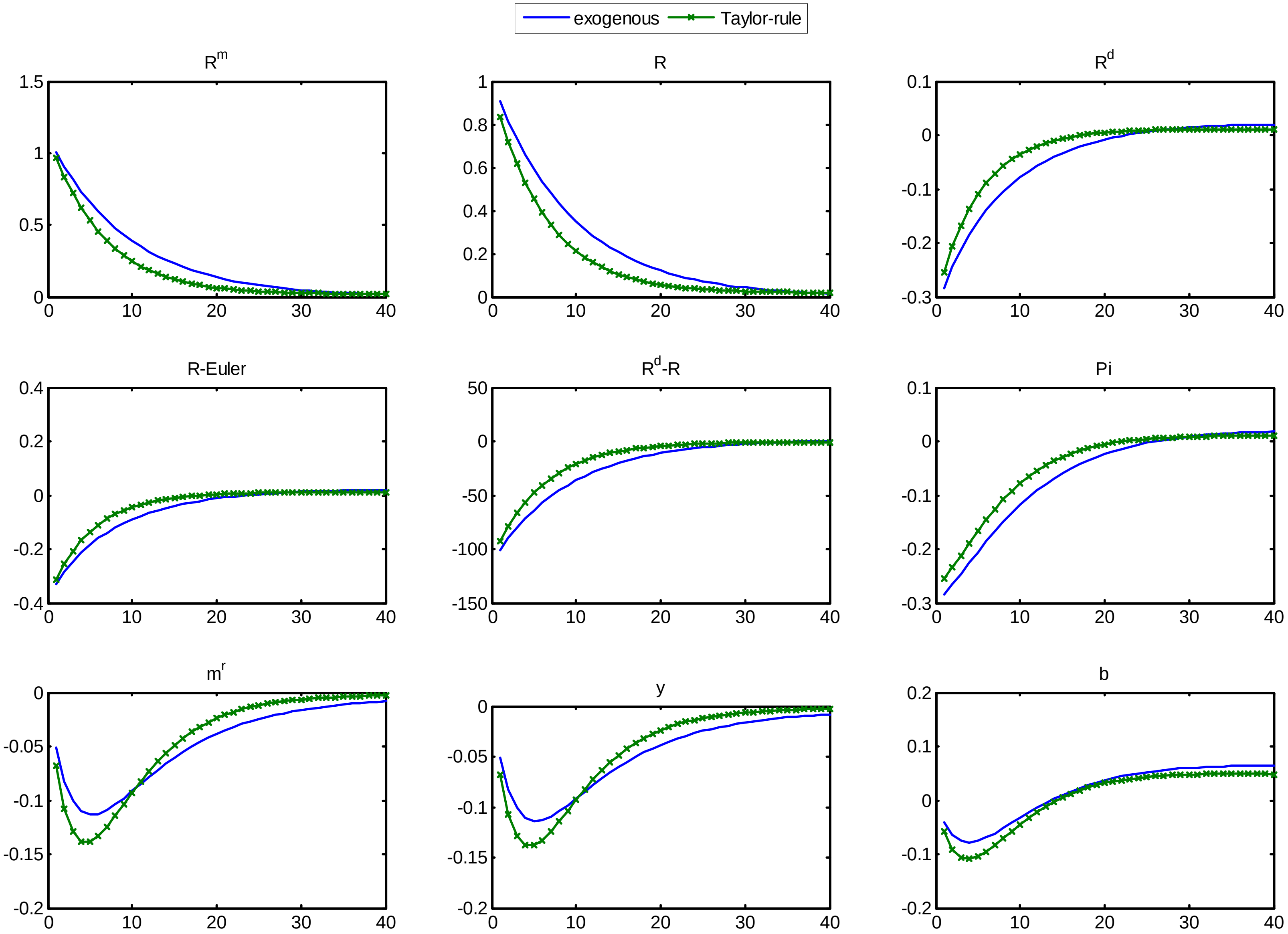
- Relation between bond price and expected price of money

$$1/R_t = E_t\left(1/R^m_{t+1}\right) + \frac{cov_t\left[\left(1/R^m_{t+1}\right), \left(u_{ct+1}/\pi_{t+1}\right)\right]}{E_t\left[u_{ct+1}/\pi_{t+1}\right]}$$

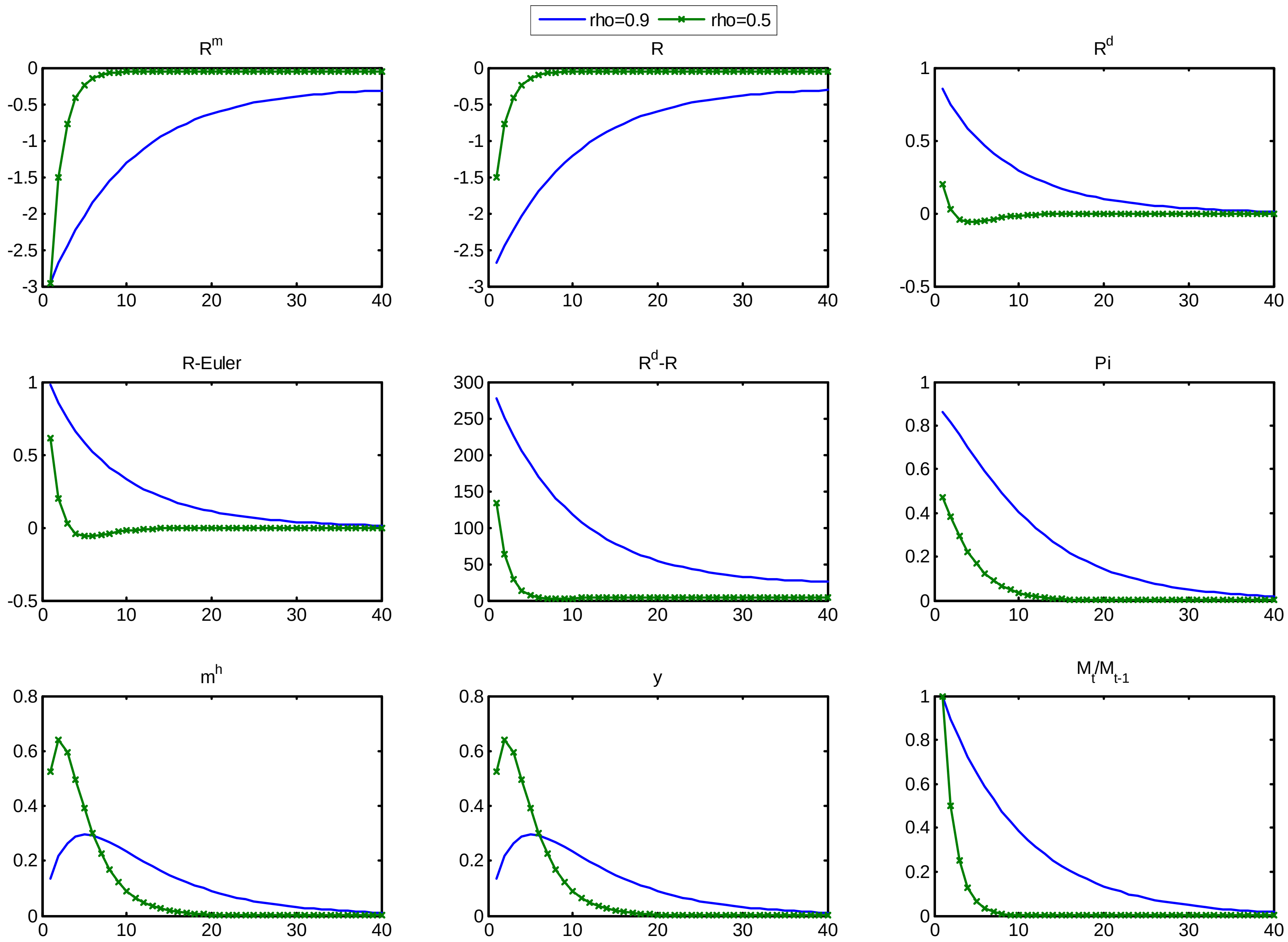
Proposition 2 *The price of government bonds is smaller than the expected future price of money,* $1/R_t < E_t(1/R^m_{t+1})$*. The average bond rate* R_t *increases with the households' relative risk aversion and with the variance of productivity shocks.*

Table 4 Spread $E_0 s_{2,t}$ for technology shocks and $\rho_\pi = 1.5$

	$\sigma = 2$, $\rho = 0$	$\sigma = 5$, $\rho = 0$	$\sigma = 2$, $\rho = 0.9$
$var(\varepsilon^a) = 0.01$	0.4 $b.p.$	1.6 $b.p.$	0.1 $b.p.$
$var(\varepsilon^a) = 0.02$	0.9 $b.p.$	3.9 $b.p.$	0.2 $b.p.$



Responses (in % dev. from st.st.) to an interest rate shock



Responses (in % dev. from st.st.) to a money supply shock for $\eta_t > 0$

Conclusions

Modeling monetary policy implementation in a more realistic way leads to the following main new results:

- short-term interest rates affected by money market / liquidity considerations, thus movements in short-term rates not directly related to aggregate demand
- evolution of “liquid” bonds (collateral) between households and central bank uncovers a new monetary policy transmission mechanism and affects the macro response to monetary shocks
- CB affects the equilibrium return on assets accepted in OMO. That return can be set as low as desired, i.e. lower than if the CB were not accepting these assets as eligible in OMO. This liquidity premium is negatively correlated with policy rate.

- the Euler-policy rates spread is negatively correlated with the policy rate
- Euler and policy rates substantially differ and are negatively correlated
- the spread between the money market and policy rates increases with aggregate risk and risk aversion
- dampened and hump-shaped output responses
- this framework helps relating monetary policy to the money market
- the framework can be used to address recent events, e.g.
  - modeling spread movements between policy and money market rates
  - modeling CB as affecting different assets yields by accepting them against cash or reserves in open market operations

ADDITIONAL SLIDES

- REE for $\psi_t > 0$ and $\eta_t > 0$ in $\{c_t, n_t, w_t, m_t^R, m_t^H, mc_t, R_t^m, R_t^d, R_t, b_t, b_t^T, \pi_t\}$

$$
\begin{aligned}
m_t^R + m_t^H &= c_t, \\
m_t^R &= \Omega m_t^H, \\
\frac{b_{t-1}}{R_t^m \pi_t} &= m_t^R + m_t^H - m_{t-1}^H \pi_t^{-1}, \\
\beta E_t \frac{u_{ct+1}}{\pi_{t+1}} &= \frac{-u_{nt}}{w_t}, \\
w_t &= mc_t \alpha a_t n_t^{\alpha-1}, \\
1/\beta &= R_t^d E_t \frac{-u_{nt+1}(n_{t+1})/w_{t+1}}{-u_{nt}(n_t)/w_t} \pi_{t+1}^{-1}, \\
R_t &= E_t u_{ct+1}(c_{t+1}) \pi_{t+1}^{-1} / [E_t \left(R_{t+1}^m\right)^{-1} u_{ct+1}(c_t) \pi_{t+1}^{-1}], \\
b_t^T &= \Gamma b_{t-1}^T \pi_t^{-1}, \\
R_t^m &= \left(R_{t-1}^m\right)^{\rho} (R^m)^{1-\rho} (\pi_t/\pi)^{\rho_\pi (1-\rho)} \exp \varepsilon_t^{\rho} \\
b_t - b_{t-1}\pi_t^{-1} &= (1-\Gamma) b_{t-1}^T \pi_t^{-1} - R_t^m \left(m_t^H - m_{t-1}^H \pi_t^{-1}\right) - (R_t^m - 1) m_t^R,
\end{aligned}
$$

and $mc_t = \frac{\varepsilon - 1}{\varepsilon}$ and $c_t = a_t n_t^{\alpha}$ for flexible prices, the tvc's, given a_t.

Table A1: Benchmark parameter values

β	γ	σ	$\Gamma = \pi$	mc	ϕ	α	s	Ω
0.984	2	2	1.0108	0.833	0.8	0.66	0.012	0.5

$\rho^{(a)}$	ρ_π	$var(\varepsilon^\rho)$	$var(\varepsilon^a)$
0.9	1.5	0.0001	0.01

Steady state values

$$R^d = 1.027 \quad (\approx 1.11 \text{ annual rate})$$

$$R = 1.015 \quad (\approx 1.06 \text{ annual rate})$$

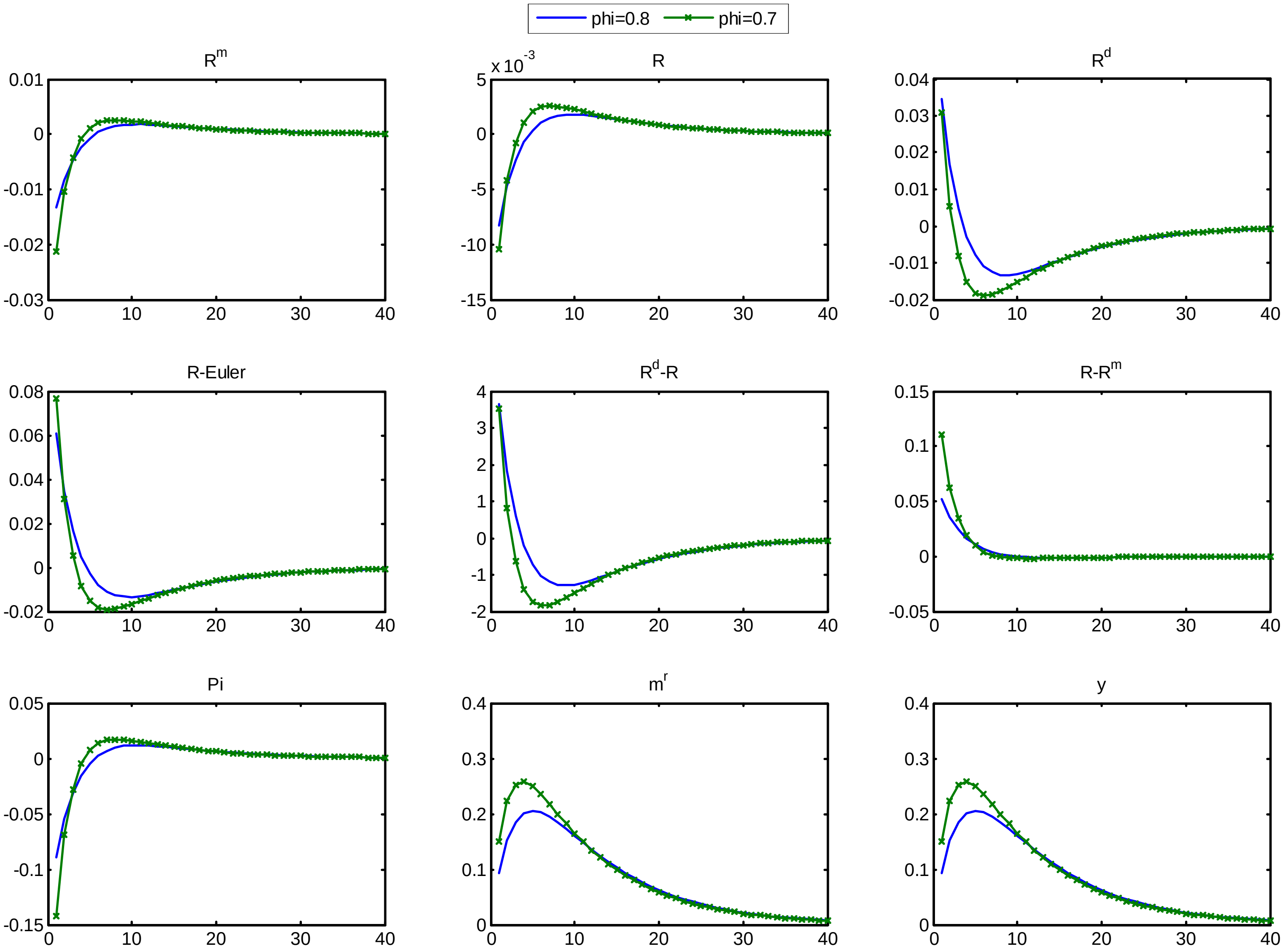
$$b/y = 0.35$$

- Details on proposition 1

$$\log\left(R_t^d/R_t\right) = -\rho\,(1+\rho)\log R_t^m - \log\beta - \left((1/2)\left(1+\rho^2\right) + (1+\rho)^2\right) var\left(\varepsilon^\rho\right)$$

- Details on proposition 2

$$E_0 R_t = \exp\left[\rho_\pi\left(\sigma\rho_\pi + \sigma - 1\right) var\left(\varepsilon_t^a\right)\right] \cdot \left(R^m\right)^{\frac{1}{\rho_\pi+1}} \pi^{-\frac{\rho_\pi}{\rho_\pi+1}}$$



Responses (in % dev. from st.st.) to a 1% productivity shock