The Economics of Helicopter Money*

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Abstract

An economy plagued by a slump and in a liquidity trap has some options to exit the crisis. We discuss helicopter money and other equivalent policies that can reflate the economy and boost consumption. Traditional helicopter money, via the joint cooperation between the treasury and the central bank, depends critically on the central bank fully guaranteeing treasury's debt. We explore some alternatives for the central bank to do helicopter money on its own, without any treasury's involvement.

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1 Introduction

"Let us suppose now that one day a helicopter flies over this community and drops an additional $1,000 in bills from the sky, which is, of course, hastily collected by members of the community. Let us suppose further that everyone is convinced that this is a unique event which will never be repeated." (Friedman, 1969)

Helicopters have been recently flying over many countries. Following the COVID-19 pandemic, the US government has approved a two trillion dollars support to the economy and the Federal Reserve has committed to unlimited quantitative easing among which purchases of the treasury's debt. The UK government has announced it would extend the size of the government's bank account at the central bank, known historically as the "Ways and Means Facility". The European Central Bank has also extended to unprecedented levels its asset purchase program. A possible implementation of Friedman's proposal is indeed to have the government doing a transfer to the citizens financed by issuing debt, which is in turn purchased by the central bank through more supply of money or reserves. Time will tell us whether this was true monetisation.[1]

In his writing, Friedman's hypothetical experiment was meant to show the effectiveness of monetary policy on inflation. It is, indeed, odd to think that the central bank cannot control the price level. At the end of the day, the Fed's liabilities define exactly what a dollar is. By virtue of this definition, the Fed has the power to print dollars at will without facing any constraint. Since the value of a dollar in terms of goods is the inverse of the price level, the Fed can really throw from the sky as many dollar bills as needed to lower the value of money and reflate the price level. Helicopter money should work!

This suggestive idea has recently received considerable attention in academia and policy circles given that central banks across the globe have lost their conventional ammunitions, having slashed the nominal interest rate down to zero. Helicopter money has been discussed as a viable option to reflate the economy (see among others Bernanke, 2002 and 2003, Galì, 2020a and 2020b, Tuner 2013, 2016).

This paper describes an economy plagued by a slump due to an adverse demand shock in which even cutting the nominal interest rate down to zero does not bring the economy to full capacity, as in the framework of Krugman (1998). Fiscal policy has only access to lump-sum transfers as effective policy tools, like at the inception of the pandemic crisis, where health-policy measures induced a contraction in labor supply that could not be offset using other tools like spending or changes in tax rates.

We study helicopter money and other alternative, and equivalent, policies that can reflate

[1] Masciandaro (2020) examines an early example of helicopter money in the Republic of Venice during the 1630 plague.

the economy, boost aggregate demand and bring the economy out of the slump.

To analyse the spectrum of available policies, it is key to understand that the central bank's liabilities (money or reserves) are special since they are free of any nominal risk, by definition. These liabilities indeed define what a dollar actually is. Therefore, the central bank can create dollars and reserves at will to pay its liabilities, without being subject to any solvency requirement. The treasury's liabilities, on the other hand, are in principle like the liabilities of any other agent in the economy. They are a promise to pay a given amount of dollars at maturity. As such, since the treasury cannot create dollars, the treasury's liabilities need to satisfy a solvency condition in order to be repaid and be nominally risk free.

The set of tools available to reflate the economy, indeed, changes depending on whether or not the treasury's liabilities are fully backed by the central bank, i.e. whether or not the special properties of the central bank's liabilities extend to the treasury's as well.

In the first case, when the treasury is backed by the central bank, helicopter money can be implemented in the traditional way. The treasury can make transfers to the private sector, or cut taxes, and finance these policies by issuing more debt. In this case, it does not really matter whether this debt is purchased by the central bank. The reason is that the treasury's debt has the same risk-free properties of the central bank's liabilities.[2] Moreover, if the central bank purchases the treasury's debt, it does not even matter whether it uses money or reserves since the economy is at the zero lower bound. However, key for the success of this combination of policies is that the treasury commit not to withdraw the short-run tax relief with higher taxes in the future. The increase in government's liabilities is therefore inflationary, lowers the real rate and stimulates aggregate demand.

The second case, in which the central bank does not back the treasury's liabilities, is quite relevant, because it describes well the current situation of the European Monetary Union where the treasuries of the several countries have to satisfy a solvency condition for the debt they issue. A tax relief today should necessarily be offset by future taxes or by default on treasury's debt. With the treasury out of the picture, however, the central bank can still rely on some policy options to reflate the economy, and all those options are equivalent to "traditional" helicopter money in terms of final outcome on prices and economic activity. We discuss three alternatives.

First, the central bank can reduce its transfers to the treasury to raise its own net worth. This policy is also inflationary. A positive central bank's net worth means that the private sector is a net debtor with respect to the definition of wealth that is relevant for its spending decisions. An increase in the central bank's net worth then corresponds to an increase in

[2] This is the case considered by Sims (2016), who rules out default on nominal public debt, based on the argument that "the government can print the money the debt promises".

the net debt position of consumers which requires an increase in the price level to ease their additional debt burden and allow demand to meet supply.

The second alternative goes through an opposite route. Instead of reducing transfers to the treasury, the central bank should write a big check to the treasury to be fully rebated to the private sector. However, two additional conditions should be fulfilled in order to reflate the economy. The first is that the transfer be so large that central bank's net worth turns negative, with the consequence that the private sector not only experiences a positive wealth effect, but it also turns into a net creditor. For a net creditor, inflation is the way to offset a positive wealth effect and restore equilibrium. The second condition is that this large, current transfer be complemented with the commitment to at least partially reverse it in the future through either lower remittances or higher seigniorage revenues and therefore a permanent higher inflation target.

The last option we discuss for the central bank to do helicopter money on its own does not need any involvement of the treasury. The central bank could do so by just writing off its credits, if any, to the private sector, therefore making a direct wealth transfer. The two additional conditions discussed above should be also satisfied in this case.

This paper is related to a recent literature that has studied liquidity trap and policy options. Krugman (1998) is our main inspiration for describing a simple model of a slump at the zero lower bound. With respect to his work, we characterise the long-run equilibrium and therefore the policies that can reflate the economy including helicopter money. Woodford (2000, 2001) is the reference for understanding the special role of the liabilities of the central bank as discussed also in recent work by Buiter (2014) and Benigno (2020). Benigno and Nisticò (2020), among others, analyse the implications of separating the treasury and the central bank for the control of inflation through central-bank balance-sheet policies.

Auerbach and Obstfeld (2005) and Buiter (2014) study experiments of helicopter drops in various models with different frictions. Along those lines, Galì (2020b) compares debt-financed versus money-financed fiscal cuts as well as the role of government purchases, and Di Giorgio and Traficante (2018) study the open-economy dimension of this comparison. Eggertsson and Woodford (2003) and Woodford (2012) stress the importance of forward guidance as an alternative way to reflate the economy out of a liquidity trap which can be equivalent in its outcome to the proposal of this work.

2 Model

We consider a simple perfect-foresight, infinite-horizon endowment monetary model in the same spirit as Krugman (1998). Time t_0 has the interpretation of the short run. The economy will be stationary after, and including, period $t_0 + 1$, which is going to be labelled the long run. There are two important features that distinguish the short from the long run: 1) prices are rigid in the short run and flexible in the long run, 2) a preference shock is low in the short run and high in the long run. For illustrative purposes, the short run lasts only one period, though we can make it longer by extending the duration of price rigidity and/or of the shock, see Section 5.

Let's see the implications of these assumptions. Consider the Euler equation

$$\xi_t U_c(C_t) = \beta(1 + i_t)\frac{P_t}{P_{t+1}}\xi_{t+1}U_c(C_{t+1}) \tag{1}$$

in which $U(\cdot)$ is the utility of consumption and $U_c(\cdot)$ its marginal utility, P_t is the price level at time t and i_t the risk-free nominal interest rate set by the central bank, β is the rate of time preference; ξ_t is a shock to preferences.

Focus first on the long run, i.e. $t \geq t_0 + 1$: prices are flexible and the preference shock is at the high level $\xi_t = \bar{\xi}$. Since prices are flexible in the long run, from $t_0 + 1$ onwards goods market clears and consumption is equal to output. Assuming a constant endowment, goods market equilibrium implies that $C_t = Y$ for each $t \geq t_0 + 1$. Section 5 extends the analysis to endogenous output. Set an interest rate policy in the long-run to target a positive rate of inflation

$$1 + i_t = \frac{1}{\beta}\Pi$$

for each $t \geq t_0 + 1$. Substituting it into (1) and using $C_t = Y$ and $\xi_t = \bar{\xi}$ we obtain that

$$\frac{P_{t+1}}{P_t} = \Pi$$

for each $t \geq t_0 + 1$: inflation is constant after $t_0 + 1$ at the level Π targeted by the central bank. What is left undetermined is the price level at time $t_0 + 1$. Let's set it, for now, at $P_{t_0+1} = \bar{P}$. We will come back to its determination later.

Consider now the short-run Euler Equation

$$\begin{aligned} U_c(C_{t_0}) &= \beta(1 + i_{t_0})\frac{P_{t_0}}{\bar{P}}\frac{\bar{\xi}}{\xi}U_c(Y) \\ &= \beta(1 + i_{t_0})\frac{P}{\bar{P}}\frac{\bar{\xi}}{\xi}U_c(Y), \end{aligned} \tag{2}$$

where in the second line we have also used the assumption that short-run prices are sticky at $P_{t_0} = P$. Assume that the distance between $\bar{\xi}$ and ξ is large enough so that, given P and

$\bar{P}$, the following inequality holds

$$\beta \frac{P}{\bar{P}} \frac{\bar{\xi}}{\xi} > 1. \tag{3}$$

Using this inequality into (2), short-run consumption falls below output at any non-negative interest rate: the economy is in a slump.

Figure 1 displays the effect of a current negative demand shock $\xi < \bar{\xi}$ on the interest rate and current consumption. In the space $(C_{t_0}, 1+i_{t_0})$, the Euler equation (2) and the zero-lower bound (ZLB) on the nominal interest rate imply a downward-sloping aggregate demand curve (AD) that dies out at $i_t = 0$. The vertical red line displays the aggregate supply curve (AS), located at the level of the constant endowment Y. Starting from a stationary equilibrium where $C = Y$ and $1 + i = \Pi/\beta$ (point A in the figure), a negative demand shock $\xi < \bar{\xi}$ shifts the AD curve to the left into AD', inducing a downward pressure on current consumption. The central bank can exploit the downward slope of aggregate demand and cut the nominal interest rate to stimulate consumption as much as possible. To restore the equilibrium in the goods market, $C_{t_0} = Y$, the central bank would need to cut the nominal rate down to $1+i_{t_0} = (\xi/\bar{\xi})(\bar{P}/(P\beta))$. However, if the size of the shock satisfies (3), the required cut in the nominal rate would violate the ZLB. As a consequence, the central bank cannot descend the AD' schedule beyond point B, where the economy is in a slump and experiences a shortage of demand:

$$\underline{C} = Y U_c^{-1}\left(\beta \frac{P}{\bar{P}} \frac{\bar{\xi}}{\xi}\right) < Y.$$

Equation (2) clarifies that the other possibility to restore the equilibrium in the goods market is to act on the future price level, reflating the economy, lowering the real rate and boosting consumption: in Figure 1, indeed, raising $\bar{P}$ shifts the aggregate demand schedule to the right into AD'' and the economy can reach equilibrium E.[3]

To understand the policies that can reflate the economy, we now move to study how the long-run price level is determined. Note indeed, that we did say something on the long-run inflation rate but not on the level of prices at time $t_0 + 1$.

Solvency of the consumer, i.e. that its debt is repaid with certainty, requires the present discounted value of expenditure not to exceed the resources available. At the optimum, the consumer exhausts all resources and its intertemporal budget constraint holds with equality.

[3]We should clarify that, although we analyse the policies to restart the economy once in a liquidity trap, the mechanisms we discuss are at work also for positive values of the nominal interest rates. In that scenario, however, the equivalence results we discuss are in general weaker.

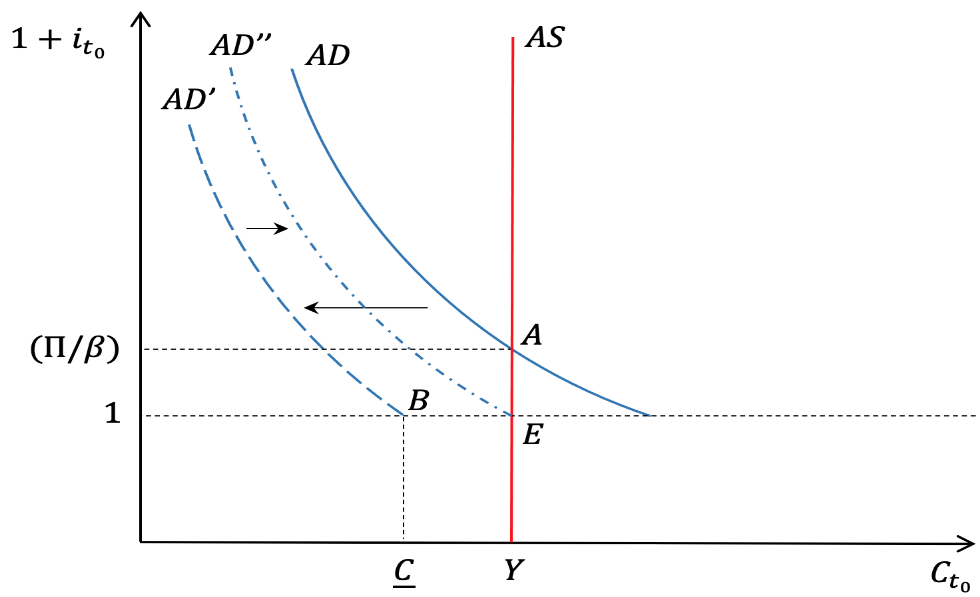

Figure 1: The effects of a negative preference shock $\xi < \bar{\xi}$: AD shifts to the left into AD' and the economy falls in a slump ($\underline{C} < Y$) due to the ZLB, unless the economy is reflated by shifting AD' to the right into AD''.

At time $t_0 + 1$ this intertemporal budget constraint is represented by

$$\sum_{t=t_0+1}^{\infty} R_{t_0+1,t} \left(C_t + \frac{i_t}{1+i_t} \frac{M_t}{P_t} \right) = \frac{W_{t_0+1}}{P_{t_0+1}} + \sum_{t=t_0+1}^{\infty} R_{t_0+1,t} \left(Y_t - \frac{T_t}{P_t} \right) \tag{4}$$

where $R_{t_0+1,t}$ is the real discount factor between period $t_0 + 1$ and a generic period t, T_t are lump-sum taxes levied by the treasury and W_{t_0+1} is the nominal wealth of the household at the beginning of time $t_0 + 1$:

$$W_{t_0+1} = B_{t_0} + (1 + \delta Q_{t_0+1}) D_{t_0} + X_{t_0} + M_{t_0}.$$

Four securities are available to households. They can save or borrow in risk-free bonds, B, and hold central bank's reserves, X; both securities pay the risk-free interest rate i. They can also save or borrow using long-term bonds, D, which pay a decaying coupon δ and sell at price Q. Finally, they can hold physical money M which does not pay any interest rate. For the services that real money balances supply, households need to pay a price, given by the foregone interest rate on bonds. The overall cost of holding real money balances is captured by the second addendum on the left-hand side of equation (4).

We can now add other results from the long-run equilibrium to simplify (4). First, equilibrium in the goods market implies that $C_t = Y$ for each $t \geq t_0 + 1$. Moreover equilibrium in the money market implies a demand of real money balances of the following form

$$\frac{M_t}{P_t} \geq L(Y, i_t) \tag{5}$$

which holds with equality whenever the interest rate is positive.[4] Real money balances are a negative function of the nominal interest rate and positively related with output. Since in the long run $i = \beta^{-1}\Pi - 1$, then $M_t/P_t = L(Y, \beta^{-1}\Pi - 1)$ for any $t \geq t_0 + 1$. Moreover the price of long-term bond satisfies the no-arbitrage condition

$$Q_t = \beta \frac{P_t}{P_{t+1}} \frac{\xi_{t+1} U_c(C_{t+1})}{\xi_t U_c(C_t)} (1 + \delta Q_{t+1}) = \frac{(1 + \delta Q_{t+1})}{1 + i_t}, \tag{6}$$

where in the second equality we used the Euler equation.

We can substitute the above restrictions in (4), using the zero interest-rate policy $i_{t_0} = 0$ – that implies $Q_{t_0} = 1 + \delta Q_{t_0+1}$ – and noting that in equilibrium $R_{t_0+1,t} = \beta^{t-t_0-1}$ to obtain

$$\sum_{t=t_0+1}^{\infty} \beta^{t-t_0-1} \left(\frac{T_t}{P_t}\right) + S(\Pi, Y) = \frac{B_{t_0} + X_{t_0} + M_{t_0} + Q_{t_0} D_{t_0}}{P_{t_0+1}}, \tag{7}$$

in which we have defined the present-discounted value of seigniorage as

$$S(\Pi, Y) \equiv \frac{\Pi - \beta}{\Pi(1-\beta)} L\left(Y, \frac{\Pi}{\beta} - 1\right).$$

The above equilibrium condition requires the long-run real value of the beginning-of-period liabilities of the whole government to be equal to the present discounted value of taxes (first term on the left-hand side) plus seigniorage revenues (the second term on the left-hand side).

We have now all the ingredients to investigate what are the policy options to reflate the economy and stimulate consumption in the short run.

To proceed we should make an important observation and distinguish two cases. The key observation is to note that the central bank has an important and exclusive power in the economy: its liabilities define the unit of account of the monetary system and therefore they are – in nominal terms – risk free by definition. This means that the central bank does not have to satisfy a solvency constraint like all other agents in the economy: its dollar obligations can always be fulfilled just by printing new dollars. In other words, while a treasury bill is redeemable for dollars, a dollar bill is only redeemable for itself.

This peculiar feature of the central bank suggests we should distinguish two cases. In the first case, appropriate institutional arrangements make the properties of the central bank's

[4] Please refer to the Appendix for a more detailed description of the model.

liabilities extend to the treasury's as well, in what we can call a consolidated view of the whole government. In the second case, the two institutions are separate and the treasury is therefore not different from any other agent in the economy since it needs to satisfy a standard solvency condition to determine the market value of its debt.[5] This second case better captures the current architecture of the EMU in which a single country's debt can be subject to default, as the Greek experience has shown.

On a modelling ground, what matters for reflating the economy are the securities that can be considered as wealth by the consumers. In the first case, when the central bank backs treasury's liabilities, they include both treasury's and central bank's liabilities. In the second case, only central bank's liabilities and the assets held by the central bank.

We start from the first case of a consolidated budget constraint for the government.

3 Central bank and treasury acting together

Consider a central bank that backs the treasury's liabilities and therefore with the risk-free property of its liabilities extending to the treasury's. Since treasury's debt is fully guaranteed by the central bank's "printing press" the treasury can run whatever fiscal policy it pleases in terms of the path of real taxes, since it does not have to necessarily satisfy an intertemporal budget constraint. As in the Fiscal Theory of the Price Level, the treasury can set the path of real taxes at $\{T_t/P_t = \tau_t\}_{t=t_0+1}^{+\infty}$ irrespective of the real value of its obligation. Equation (7), which is an equilibrium condition but not a solvency constraint, can determine the price level P_{t_0+1} at, let's say, $\bar{P}$. It should be read in the following way. It is not the left-hand side of (7) that necessarily adjusts to back the right-hand side – i.e. the real value of the outstanding government's nominal liabilities at any equilibrium P_{t_0+1}. The other way round, indeed: long-run prices adjust to satisfy the equilibrium condition, given monetary and fiscal policies that determine the left-hand side of (7) and given the outstanding government's nominal liabilities at time $t_0 + 1$:

$$\bar{P} = \frac{B_{t_0} + X_{t_0} + M_{t_0} + Q_{t_0} D_{t_0}}{\sum_{t=t_0+1}^{\infty} \beta^{t-t_0-1} \tau_t + S(\Pi, Y)}. \tag{8}$$

To complete the analysis, consider the flow budget constraint of the government in period t_0

$$B_{t_0} + X_{t_0} + M_{t_0} + Q_{t_0} D_{t_0} = B_{t_0-1} + X_{t_0-1} + M_{t_0-1} + (1 + \delta Q_{t_0}) D_{t_0-1} - T_{t_0}, \tag{9}$$

[5] See Buiter (2014) and Benigno (2020) for a discussion of this point.

where on the left-hand side we have used the fact that the nominal interest rate is at the ZLB in the current period $i_{t_0} = 0$.

Equation (8) shows the alternative policy options to reflate the economy. Before describing them, note that the Fiscal Theory of the Price Level is essential for what follows. A Ricardian fiscal policy would lead to indeterminacy of the price level and, therefore, may not allow the policymaker to control long-run prices and boost the economy.

The first option to reflate the economy is to raise the numerator of (8), *ceteribus paribus*, the so called "helicopter money" , where the government (treasury or central bank) increases permanently the long-run nominal liabilities – namely B_{t_0}, X_{t_0}, M_{t_0} or D_{t_0} – to finance a tax cut in the short run. Since the short-run nominal interest rate is zero, all these possibilities are equivalent, as implied by (9).[6] Indeed, given that all the government's liabilities have the special properties of the central bank's, B_{t_0}, X_{t_0}, M_{t_0} or D_{t_0} are always paid in full since they are guaranteed by the "printing press" of the central bank without any need to raise taxes or seigniorage revenues. And, indeed, taxes and seigniorage should not move (proportionally) for an increase in government debt to produce an effect on long-run prices, as equation (8) shows.

Moreover, equation (9) clarifies that the increase in government liabilities outstanding at $t_0 + 1$ can be generated by a tax cut at t_0 and therefore a larger current primary deficit. This larger deficit can equivalently be financed issuing either short-term or long-term treasury's debt, which can equivalently be held by either the private sector or the central bank. In the former case B_{t_0} or D_{t_0} increase for given X_{t_0} and M_{t_0}, while in the latter case the opposite occurs, as the central bank raises its liabilities – either money or reserves – to absorb the new issuance of treasury's debt, leaving unchanged the stock of debt held by the private sector (B_{t_0} and D_{t_0}). In the latter case, it does not really matter whether the central bank holds permanently the treasury's debt or writes it off, as discussed in Buiter (2014) and Galì (2020a) and (2020b).

For all these policy options to succeed, equation (8) clarifies that it is important that the denominator does not change (at least not proportionally): the treasury should therefore commit to not undo the short-run tax relief.[7]

It is useful to visualise results using a simple *AD–AS* logic. To this end, we can use the equilibrium condition (4) and exploit some simplifications on preferences as outlined in the Appendix (namely log utility in consumption and real money balances), to write long-run

[6] Some equivalence results are going to break at positive short-run interest rate, but not the overall argument on the general effectiveness of the policies proposed.

[7] This is therefore an example of "unbacked fiscal expansion", in the words of Jacobson, Leeper and Preston (2019).

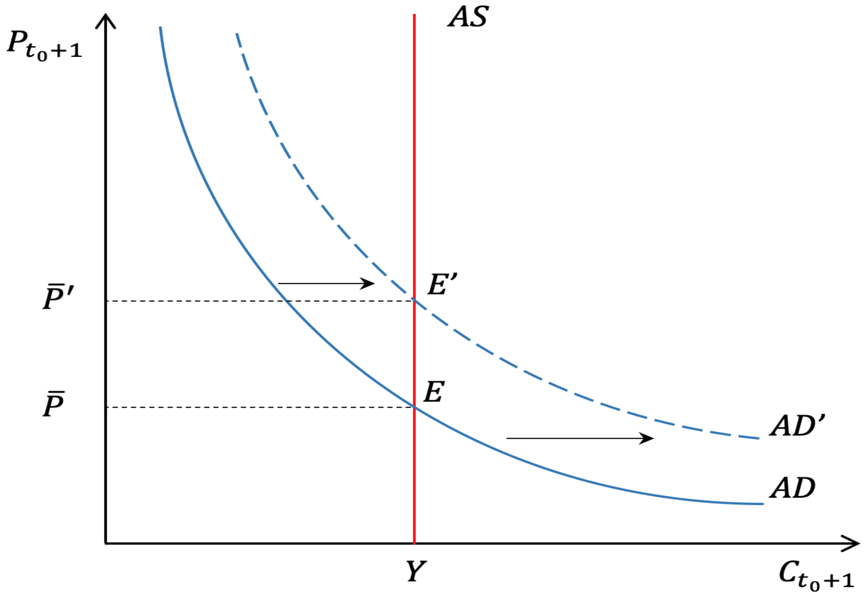

Figure 2: Reflating the economy when the government faces a consolidate budget constraint.

consumption as

$$C_{t_0+1} = \frac{1-\beta}{1+\theta} \left\{ \frac{B_{t_0} + X_{t_0} + M_{t_0} + Q_{t_0} D_{t_0}}{P_{t_0+1}} + \sum_{t=t_0+1}^{\infty} R_{t_0+1,t} \left(Y - \tau_t \right) \right\}, \tag{10}$$

for some positive preference parameter θ. This equation can be interpreted as a consumption demand equation relating long-run consumption to long-run prices. However, the channel through which prices affect consumption is not a conventional one since it acts through the financial wealth held by the agent – the one that is irredeemable. Assuming that the private sector is a net creditor with respect to the government (i.e. $B_{t_0} + X_{t_0} + M_{t_0} + Q_{t_0} D_{t_0} > 0$), then equation (10) implies a negative relationship between long-run prices and consumption. For a creditor, indeed, an increase in the price level reduces the real value of his/her assets pushing consumption down. This relationship is plotted in Figure 2 as an AD equation together with the AS equation of constant long-run output.

Consider now an increase in the government's nominal liabilities at time t_0. Since the agent is a net creditor, this raises the nominal financial wealth that agents carry into period $t_0 + 1$ and creates an excess demand of goods at the initial price level: the demand curve shifts to the right into AD'. In order for consumption to fall back to the level of the constant endowment, such excess demand stimulates an increase in the price level that reduce the real value of the financial assets held by the consumer and restore equilibrium in E'.[8]

[8]Equation (8) suggests two alternative policy options to reflate the economy, which work through a reduction in the denominator. The first alternative is a treasury's commitment to lower real taxes in the

4 Central bank acting alone

Consider now the case in which the central bank does not directly back treasury's liabilities. As we have discussed, this is a case of practical interest in Europe where the central bank and national treasuries are not directly linked. In this case, the treasury should be subject to a standard solvency condition of the following form

$$\sum_{t=t_0+1}^{\infty} R_{t_0+1,t}\left(\frac{T_t}{P_t}+\frac{T_t^C}{P_t}\right)=\frac{B_{t_0}^T}{P_{t_0+1}}+\frac{(1+\delta Q_{t_0+1})D_{t_0}^T}{P_{t_0+1}}, \tag{11}$$

at *any* equilibrium price, in which B^T and D^T are total short and long-term treasury's debt, respectively. Given the remittances received from the central bank, T_t^C, the treasury should passively adjust taxes in a way to back its short- and long-term liabilities at any equilibrium prices. If taxes are not adjusted, then treasury should default at least partially on its debt obligation and the above condition will hold at any equilibrium price with the right-hand side adjusting for the recovery rate on debt. This implies that by no means treasury's debt can be considered wealth for the private sector, since any increase in debt should be offset by either a corresponding increase in the present discounted value of future taxes or a (partial) default on it.[9]

Note that in equilibrium the total short-term debt issued by the treasury is held by either the central bank or the private sector, implying

$$B^T=B^C+B,$$

and analogously for long-term debt

$$D^T=D^C+D,$$

where D^C may also include debt issued by the private sector. Using these equilibrium conditions together with (11) into (4), we obtain the relevant equilibrium condition to determine the price level at time t_0+1:

$$\sum_{t=t_0+1}^{\infty} \beta^{t-t_0-1}\left(\frac{T_t^C}{P_t}\right)=S\left(Y,\Pi\right)+\frac{Q_{t_0}D_{t_0}^C+B_{t_0}^C-X_{t_0}-M_{t_0}}{P_{t_0+1}}, \tag{12}$$

long run, given an unchanged path of liabilities carried from t_0. The second alternative is a central bank's commitment to lower the present discounted value of seigniorage revenues by changing the inflation target Π. The sign of the required change in Π is ambiguous, and depends on whether seigniorage evaluated at the target rate of inflation, Π, is increasing or decreasing in Π.

[9] Ricardian equivalence holds given the lump-sum nature of taxes.

having used $R_{t_0+1,t} = \beta^{t-t_0-1}$, the seigniorage function $S(Y,\Pi)$ and set the interest rate at time t_0 to zero.

Equation (12) emphasizes two implications with respect to the previous case that are key to understand the results that will follow. First, the relevant definition of private wealth is now mirrored by the net financial position of the central bank alone $(Q_{t_0}D^C_{t_0}+B^C_{t_0}-X_{t_0}-M_{t_0})$ instead of the whole government's, since the treasury's liabilities are no longer wealth for the private sector. Second, what matters for price determination is now the path of central bank's remittances, instead of taxes. As in the fiscal theory of the price level, the specification of the remittances policy is critical for determining the price level in the long-run and reflating the economy. A real transfer policy is sufficient.[10] Assume that $\{T^C_t/P_t = \tau^C_t\}^{+\infty}_{t=t_0+1}$, then we can write

$$\sum_{t=t_0+1}^{\infty} \beta^{t-t_0-1}\tau^C_t = S(Y,\Pi) + \frac{Q_{t_0}D^C_{t_0}+B^C_{t_0}-X_{t_0}-M_{t_0}}{\bar{P}}.$$

This is again an equilibrium condition, not a solvency constraint since X_{t_0} and M_{t_0} are free of any nominal risk, which then determines the long-run price level $\bar{P}$ at

$$\bar{P} = \frac{N_{t_0}}{\sum_{t=t_0+1}^{\infty} \beta^{t-t_0-1}\tau^C_t - S(Y,\Pi)} \tag{13}$$

where central bank's net worth is defined as[11]

$$N_t \equiv Q_t D^C_t + \frac{B^C_t - X_t}{1+i_t} - M_t. \tag{14}$$

To complete the analysis, note that the law of motion of net worth is given by

$$N_t = N_{t-1} + \Psi_t - T^C_t, \tag{15}$$

where central bank's profits are

$$\Psi_t \equiv i_{t-1}(N_{t-1}+M_{t-1}) + (r_t - i_{t-1})Q_{t-1}D^C_{t-1},$$

having defined as r_t the return on long term securities, i.e. $1+r_t = (1+\delta Q^C_t)/Q^C_{t-1}$.

Assume first that nominal net worth is positive. For the price level in (13) to be positive, the denominator of (13) should be also positive. The central bank – and only the central bank – has now several policy options available to reflate the economy. First, it could act on the

[10] The Appendix discusses price determination under a nominal remittances policy and an active interest-rate policy. The equilibrium in which $P_{t_0} = P^*$ coesists with a non-monetary equilibrium.

[11] In the numerator of equation (13) we use definition (14) and the fact that at time t_0 the nominal interest rate is at the ZLB.

numerator of (13), by raising its net worth, *ceteribus paribus.* This can be accomplished by reducing short-run transfers to the treasury, as shown by (15), which implies higher current taxes for the private sector.

To understand the intuition behind this counter-intuitive mechanism, notice that treasury's debt is not at all considered wealth by the private sector since it is paid by future taxes levied on the private sector itself. Consumption demand, in this case, can be written by combining (4) and (11), under the simplifying preference specification used in the Appendix, as

$$\bar{C} = \frac{1-\beta}{1+\theta}\left\{-\frac{Q_{t_0}D_{t_0}^{C} + B_{t_0}^{C} - X_{t_0} - M_{t_0}}{\bar{P}} + \sum_{t=t_0+1}^{\infty} R_{t_0+1,t}(Y+\tau_t^{C})\right\}. \tag{16}$$

This demand function clarifies that the net asset position of the consumer, which can be considered as wealth, mirrors that of the central bank only: if the central bank's net worth is positive ($N_{t_0} \equiv Q_{t_0}D_{t_0}^{C} + B_{t_0}^{C} - X_{t_0} - M_{t_0} > 0$) then the private sector is a net debtor with respect to the wealth position that matters for its consumption. The demand function has now a different shape simply because the consumer has a negative position with respect to financial wealth. For a net debtor, indeed, an increase in the price level reduces the real value of his/her obligations, thereby pushing up consumption: equation (16) now implies a positive relationship between long-run consumption and the price level (AD schedule in Figure 3).

Consider now an increase in the net asset position of the central bank, produced by a cut in remittances at t_0 (and therefore a short-run monetary contraction). In a specular way, this implies a deterioration of the net debt position for the private sector and a negative wealth effect which induces a fall in demand, at the initial price level, and an excess supply of goods: the AD schedule shifts to the left into AD'. Since the agent is a net debtor, therefore, in order for the constant endowment to be entirely absorbed by consumption, such excess supply now stimulates a rise in the price level that can ease the real debt burden on consumers and stimulate their demand up to the point where it is equal to supply (i.e. $\bar{C} = Y$) in the new equilibrium E'.[12] In Section 5, we show that this result does not depend on some of the simplifying assumptions of this Section, namely a two-period model with exogenous output, fully rigid prices in the short run and flexible in the long run. Indeed, this finding will extend

[12] The alternative policy options to achieve the same allocation work through changing the denominator of (13). The central bank could commit to reduce the present-discounted value of real remittances transferred in the long run, which at the end means higher taxes for the households. But the mechanism is similar as above, since the reduction in the present-discounted value of net income for the households deteriorates their overall wealth position at the initial price level. Therefore an increase in the price level is required in the new equilibrium to reduce the real value of the financial liabilities of the household and compensate the fall in human wealth. By the same logic, committing to an increase in future seigniorage revenues can now reflate the economy.

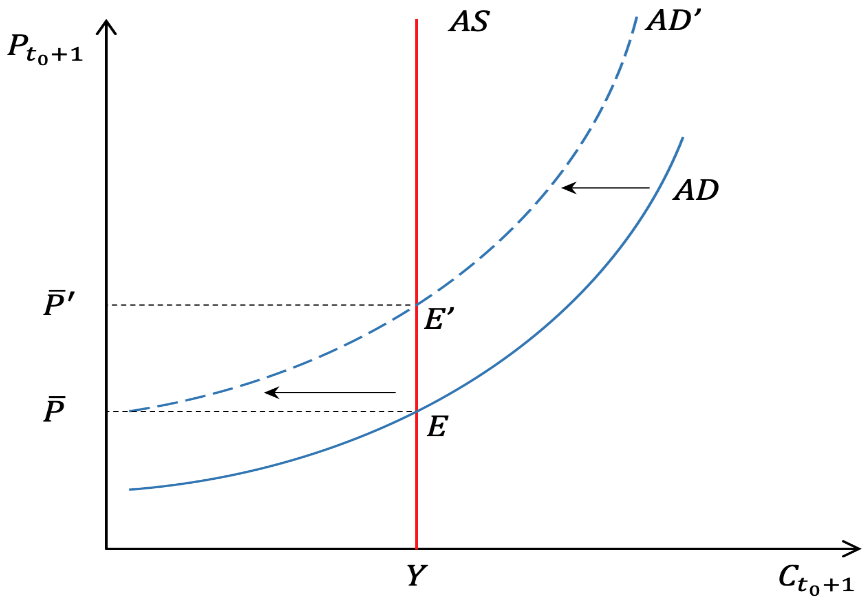

Figure 3: Reflating the economy when the central bank acts alone: the case of positive net worth.

unchanged to the benchmark New-Keynesian model.

To get an intuition of this apparently counterintuitive result, we notice that it echoes a popular proposition in monetary economics, the "unpleasant monetary arithmetic" of Sargent and Wallace (1984). There, too, a monetary contraction in the short run ends up producing more inflation eventually. We think the parallel is interesting because the mechanism is technically similar, while its economic significance is very different. In Sargent and Wallace (1984), the underlying key condition for the "unpleasant" result is fiscal dominance and an active fiscal policy: the monetary tightening in the short run sets the public debt on an an upward diverging path; then, fiscal dominance and the need to restore solvency of the government in the long run imply that, eventually, the growth rate of money needs to increase in order to finance the fiscal deficit and offset the net financial position of the government. Importantly, in Sargent and Wallace (1984) the budget constraint of the public sector is consolidated, which rationalizes the need for money to adjust eventually: the central bank is backing the treasury's liabilities.

Here, instead, fiscal policy is passive, so we are in a monetary dominance world. And the central bank faces an independent and separate budget constraint. Where is the similarity then? Although fiscal policy is passive, the remittance policy is instead active. Indeed, the monetary dominance in this setting includes two sub-regimes: one in which remittances are passive to ensure stationary real net worth for any price level (we might call it "interest-rate dominance"), and one in which they are active, meaning they are unrelated to the path of net worth (we might call it "balance-sheet dominance"). In the latter case, the real net

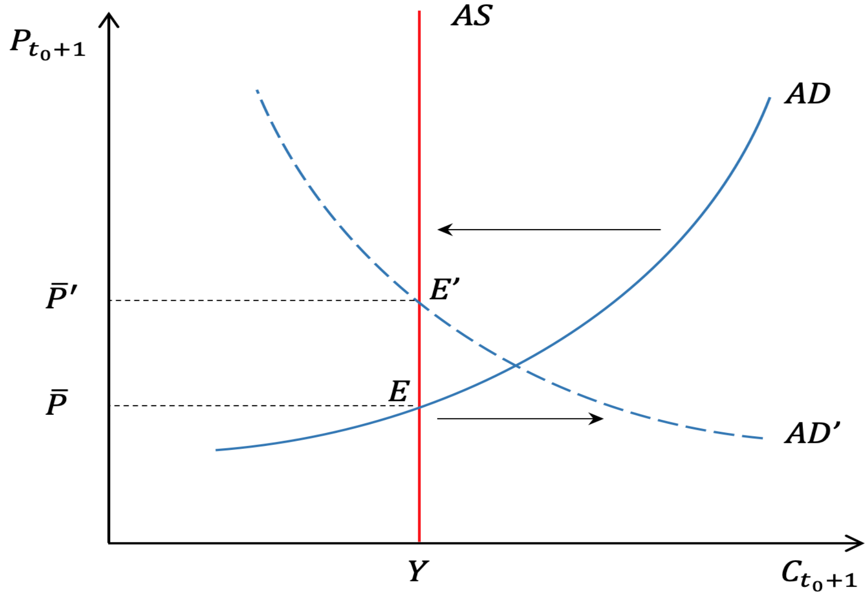

Figure 4: Reflating the economy when the central bank acts alone: "helicopter money" through negative net worth.

worth will follow an analogously diverging path, requiring the price level to adjust to restore equilibrium. In this case the short-run monetary tightening sets the real net worth – at the initial price level – on an upward-sloping and diverging path, implying that the net financial position of the private sector keeps deteriorating. Long-run solvency of the private sector then requires, eventually – i.e. at $t_0 + 1$ – that the central bank reverts the tight money through higher nominal remittances that support a higher price level (consistently with its exogenous real remittances policy) and restore solvency of the private sector in real terms.

Anyhow, equations (13) and (16) suggest that there are other tools available to the central bank, which would also work and relate more directly to policy options discussed in the literature, such as "helicopter money". The central bank makes in this case a sufficiently large transfer to the private sector financing it through higher seigniorage in the future. The transfer should be large enough to turn its net worth negative, the numerator on the right-hand side of (13). As implied by equations (14) and (15), there are two ways to turn N_{t_0} negative. The first is to make a direct transfer by writing off some of the assets held from time $t_0 - 1$, the ones issued by the private sector. This has a direct positive wealth effect on the private sector without any involvement by the treasury. The second is to make an indirect transfer by increasing the remittances to the treasury or by writing off part of treasury's debt held in its portfolio. The larger resources obtained by the treasury can be rebated to the private sector through a matching tax cut, to satisfy equation (11).[13]

[13] See Benigno and Nisticò (2020) for proof that a tax rule satisfying restriction (11) requires the treasury to rebate to the private sector any remittances received by the central bank, period by period.

The private sector, therefore, experiences a positive wealth effect in both cases. It is important to emphasize, however, that the key mechanism behind this version of "helicopter money" relies on turning the private sector into a net creditor with respect to the financial securities that can be considered as wealth. Under this condition, indeed, the excess demand of goods induced by the positive wealth effect is able to stimulate an increase in the price level that reduce the real value of the private net asset position and allow demand to meet supply. On the contrary, a positive wealth effect on the private sector that is not so large to make it a net creditor would not work in reflating this economy. An increase in the price level, indeed, would improve the financial position of the private sector and exacerbate the excess demand even further. In this case, instead, a fall in the price level is required in order to worsen the net debt position and absorb the excess demand.

Equation (13), moreover, shows that the proposed policies should be complemented with further actions in order for the price level to be positive and consistent with an equilibrium. Indeed, if the numerator in (13) turns negative, so should the denominator. Therefore, it should be that

$$\sum_{t=t_0+1}^{\infty} \beta^{t-t_0-1} \tau_t^C < S(Y, \Pi),$$

which can be obtained by either lowering the present-discounted value of the remittances in the long run or by raising seigniorage revenues through an increase in the inflation target, if seigniorage is on the left side of the Laffer curve.

Figure 4 shows that these policies work in a similar way as the "helicopter money" experiment. Like the latter, indeed, the central bank's transfer at t_0 is reflected into an improvement in the long-run net financial position of the private sector and implies a positive wealth effect that boosts aggregate demand in $t_0 + 1$ and shifts the AD schedule to the right. However, in this case, key is that the improvement in the net financial position of the private sector is large enough to turn it into a net creditor. Indeed, the reason why upward pressures on aggregate demand turn out to be inflationary now (as opposed to before) is that turning the private sector into a net creditor not only shifts the AD schedule to the right, but it also flips it into a downward-sloping curve. It is precisely this switch in the slope of the AD schedule that allows the central bank to reflate the economy through an upward pressure on long-run aggregate demand: since the economy is already at full capacity, indeed, the surge in demand stimulates an increase in the price level that reduces consumer's real wealth and brings consumption back the output level.

We now compare the policy of writing off central bank's assets discussed in this Section with the proposal of Buiter (2014) and Galì (2020a) and (2020b). In their case, the mechanism runs as follows. First, the treasury lowers taxes financing the cut with newly issued debt purchased by the central bank through an increase in its liabilities (reserves or money). Then, the central bank writes off the treasury's debt or, equivalently, it rolls it over permanently. What is important in their proposal is the lowering of taxes financed at the end by the increase in central bank's liabilities. In our proposal, instead, there is no need to increase central bank's liabilities (money or reserves) and the treasury can be completely uninvolved as long as the central bank writes off private securities from its balance sheet. However, in our model, it is critical that the central bank's intervention be large enough that its net worth becomes negative, unlike in Buiter (2014) and Galì (2020a) and (2020b).

5 Robustness

In this Section we discuss the robustness of our analysis along two dimensions. The first is the length of the short run and, therefore, whether results depend on the duration of fixed prices. Let us consider a short run lasting two periods, t_0 and $t_0 + 1$, instead of one as in the benchmark case. The long run is therefore shifted forward in period $t_0 + 2$. The analysis can be easily generalized to a longer short run. As before, in the short run, prices are sticky, therefore $P_{t_0} = P_{t_0+1} = P$ and the preference shock is at the low level, $\xi_{t_0} = \xi_{t_0+1} = \xi$; in the long run, the preference shock is at the high level and therefore $\xi_t = \bar{\xi}$ for each $t \geq t_0 + 2$. Inflation is on target after $t_0 + 2$ and the price level at time $t_0 + 2$ is $\bar{P}$. By writing the Euler equation at time t_0 and using the simplifying assumption of log consumption utility we get

$$C_{t_0} = \frac{1}{\beta(1 + i_{t_0})} C_{t_0+1}$$

in which we have used the two assumptions that $P_{t_0} = P_{t_0+1} = P$ and $\xi_{t_0} = \xi_{t_0+1} = \xi$. At $t_0 + 1$ the Euler equation instead reads as

$$C_{t_0+1} = \frac{1}{\beta(1 + i_{t_0+1})} \frac{\xi}{\bar{\xi}} \frac{\bar{P}}{P} Y$$

where we used the appropriate specifications of prices and preference shocks between short and long run and we set $C_{t_0+2} = Y$. Combining the above two equations we get:

$$C_{t_0} = \frac{1}{\beta^2(1 + i_{t_0})(1 + i_{t_0+1})} \frac{\xi}{\bar{\xi}} \frac{\bar{P}}{P} Y.$$

Under the assumption that

$$\beta^2 \frac{P}{\bar{P}} \frac{\bar{\xi}}{\xi} > 1,$$

we can replicate the analysis of the previous sections and note that, even in case interest rates are zero in both periods t_0 and t_0+1, consumption remains below output in the short tun. Having set these short-term rates to zero, the only way policymakers can raise C_{t_0} is by lifting off the long-run price level $\bar{P}$. Therefore the analysis will follow similar lines of previous sections where what matters for the determination of the long-run price level is the government's asset/debt position that will be carried in period t_0+2.[14]

The second robustness exercise relaxes the assumption that prices are completely sticky in the short run and the consequent implication that goods market may not clear. It also considers endogenous output. We borrow in this case the framework from the benchmark New-Keynesian model with price rigidities à la Calvo, see Galì (2008) or Woodford (2003).

In a log-linear approximation and in a perfect foresight equilibrium, the AD equation is given by

$$\hat{Y}_t = \hat{Y}_{t+1} - \sigma[\hat{\imath}_t - r_t^n - (\pi_{t+1} - \pi)], \tag{17}$$

where $\hat{Y}_t \equiv \log(Y_t/\bar{Y})$ and $\hat{\imath}_t \equiv i_t - \bar{\imath}$, where $\bar{\imath}$ is the steady-state level of the nominal interest rate, while r_t^n is the natural rate of interest; π_{t+1} and π are the logs of inflation rate at time $t+1$ and of the inflation target, respectively; σ is the intertermporal elasticity of substitution in consumption. The AS equation is given by

$$\pi_t - \pi = \kappa\hat{Y}_t + \beta(\pi_{t+1} - \pi) \tag{18}$$

in which κ is a combination of parameters and captures the slope of the AS equation. We focus on the case in which the treasury follows a passive fiscal policy implying that the solvency condition is satisfied at all times and public debt is therefore considered risk free. The relevant equilibrium condition for price determination is, thus, equation (13). Under the simplifying assumption that there is no long-term debt, we can write it as

$$\frac{(1+i_{t-1})N_{t-1}}{P_t} = \sum_{T=t}^{\infty} R_{t,T}\frac{T_T^C}{P_T}. \tag{19}$$

Note that in the standard New-Keynesian model money is not held in equilibrium at positive interest rate and therefore seigniorage revenues are always zero and nominal profits are simply $\Psi_t = i_{t-1}N_{t-1}$. Writing the first-order approximation of the above equation, we obtain

$$\hat{\imath}_{t-1} + \hat{n}_{t-1} - (\pi_t - \pi) = (1-\beta)\hat{\tau}_t^C + \beta\hat{n}_t \tag{20}$$

[14] The only, obvious, difference with the previous analysis is that the negative preference shock needs now to be stronger than in Section 2 in order for the ZLB to bind.

in which $n_t = N_t/P_t$ and all the other variables have been previously defined.

The set of equations (17)–(20) can be used to determine inflation and output, which is now endogenous and coincides in equilibrium with consumption. Coherently with Section 2, we make the following assumptions: a) at time t_0 the economy is hit by a preference shock that brings the natural rate of interest down to a negative number, i.e. $r^n_{t_0} = -r < 0$ while no shock occurs afterward; b) after and including time t_0+1 the central bank sets a constant interest rate, at the steady-state target, i.e. $\hat{\imath}_t = 0$, while sets it to zero at time t_0, i.e. $\hat{\imath}_t = -\bar{\imath}$ where $\bar{\imath}$ is the steady-state log interest rate; c) the remittances policy is constant at the steady-state level after and including period t_0+1. Therefore, we extend the example of Figure 3 to the benchmark New-Keynesian model.

As shown in the Appendix, the model can be solved backward to obtain that output, inflation and real net worth are the following linear functions of $\hat{n}_{t-1}$

$$(\pi_t - \pi) = \psi \hat{n}_{t-1}, \qquad \hat{Y}_t = \phi \hat{n}_{t-1}, \qquad \hat{n}_t = \lambda \hat{n}_{t-1}, \tag{21}$$

for each $t \geq t_0+1$, in which λ is a parameter with $0 < \lambda < 1$ and $\psi = 1 - \beta\lambda \in (0,1)$ and $\phi = \psi\sigma\lambda/(1-\lambda) > 0$. Using this solution into equations (17)–(18) at time t_0, we can then solve for the endogenous variable at time t_0 to obtain

$$p_{t_0} = \varpi(p_{t_0-1} + \pi) + (1-\varpi)(\hat{N}_{t_0} - \bar{\imath}) - \varpi\kappa\sigma(r - \bar{\imath}) \tag{22}$$

$$\hat{Y}_{t_0} = \varpi(\phi + \sigma\psi)(\hat{N}_{t_0} - \bar{\imath}) - \varpi(\phi + \sigma\psi)(p_{t_0-1} + \pi) - \varpi\sigma\left(1 + \beta\psi\right)(r - \bar{\imath}) \tag{23}$$

in which

$$0 < \varpi = \frac{1}{1 + \kappa(\phi + \sigma\psi) + \beta\psi} < 1.$$

The above equations show that, at equilibrium, both the log of the price level, p_{t_0}, and the log-deviation of output from steady state, $\hat{Y}_{t_0}$, at time t_0 are positively related to the log-deviation of nominal net worth from the initial steady state, $\hat{N}_{t_0} \equiv \log(N_{t_0}/\bar{n})$. Therefore, the analysis of Section 4 applies: the central bank can reflate the economy by raising its net worth at time t_0 which can be achieved through lower remittances at the same time. Since prices are not completely rigid at t_0, this policy increases not only output but also the price level at the same time. Moreover, since the price-level response to increases in *nominal* net worth $\hat{N}_{t_0}$ is less than unitary, i.e. $\varpi > 0$, current *real* net worth $\hat{n}_{t_0}$ also rises, implying that inflation overshoots its target at time t_0+1, as shown by (21), to converge back to it in the long run. This confirms that the implications from the previous section extend to the standard New Keynesian framework as well: for a given path of future remittances, an increase (decrease) in the current nominal net worth passes through into an upward (downward) pressure on the future price level, reflating the economy and stimulating current consumption.

On the other hand, if prices were always perfectly flexible – i.e. $\kappa \to \infty$ implying $\varpi = 0$ and $\varpi\kappa = 1/(\phi + \sigma)$ – the central bank's intervention on *nominal* net worth would impact current prices one for one (leaving therefore *real* net worth unchanged), and could completely undo the deflationary effect of the negative preference shock in the current period with no ripples in the future, while neither the shock nor the central bank's action would have any effects on output.

6 Conclusion

This paper studies the economics behind policies available to reflate an economy out of a slump. We discuss a set of policy actions that are all equivalent to the standard specification of "helicopter money", and characterize the alternative mechanisms at work depending in particular upon specific institutional arrangements between the central bank and the treasury.

We have kept our model as simple and tractable as possible. Several extensions can address the limitations of our analysis. First, a more elaborate dynamic extension could be helpful to understand the effectiveness of policies even in the medium run, and would allow to capture the endogenous duration of the ZLB policy depending on the policies used to reflate the economy, along the lines of Eggertsson and Woodford (2003). This extension could interestingly break the equivalence between some of the policies we discuss. A second important assumption of our framework is the lump-sum nature of transfers or taxes: this is motivated by the observation that fiscal policy can also be in a trap under certain shocks that bound the availability of effective tools to just lump-sum transfers.[15] This assumption diminishes the effectiveness of fiscal policy, when the central bank does not fully back its liabilities, because Ricardian equivalence holds. Assuming distortionary taxes or productive public spending can, in general, give more role to fiscal policy to boost the economy out of the slump, as discussed by Eggertsson (2011). It would be interesting to compare the effectiveness of alternative fiscal tools with those explored in this work.

[15] This was the case during the Great Lockdown.

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A Appendix

A.1 General Model

In this section, we describe the features of the general model used in the main text.

The representative household has the following objective function

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} \xi_t \left[U(C_t) + v\left(m_t\right) \right] \tag{A.1}$$

where C is final private consumption, ξ is an inter-temporal preference shock affecting the discount rate, and $m \equiv M/P$ denotes real money balances. Utility from consumption $U(\cdot)$ is increasing and concave, with $U_C(\cdot) > 0$ and $U_{CC}(\cdot) < 0$. Utility from real money balances is also increasing and concave, with $v_m(\cdot) \geq 0$ and $v_{mm}(\cdot) \leq 0$; to account for the zero-lower bound in the nominal interest rate, we assume the existence of a satiation level in real money balances $\bar{m}$, such that $v_m(m_t) = 0$ for $m_t \geq \bar{m}$.

The household's budget constraint is

$$P_t C_t + M_t + \frac{B_t + X_t}{1 + i_t} + Q_t D_t \leq P_t Y + M_{t-1} + B_{t-1} + X_{t-1} + (1 + \delta Q_t) D_{t-1} - T_t, \tag{A.2}$$

where Y is a constant endowment, M is nominal currency, B and X are nominal short-term bonds and central bank's reserves, respectively, both carrying the nominal interest rate i, D is long-term bonds, selling at nominal price Q and paying a geometrically decaying coupon δ, P is the price level and T are lump-sum taxes levied by the Treasury.

Let λ_t the Lagrange multiplier on the budget constraint at time t, the first-order optimality conditions with respect to consumption and nominal currency are

$$\xi_t U_C(C_t) = \lambda_t P_t \tag{A.3}$$

and

$$\xi_t v_m(m_t)/P_t + \beta \lambda_{t+1} = \lambda_t. \tag{A.4}$$

First-order conditions with respect to B (or X) and D are, respectively

$$\beta(1 + i_t)\lambda_{t+1} = \lambda_t \tag{A.5}$$

and

$$\beta(1 + \delta Q_{t+1})\lambda_{t+1} = \lambda_t Q_t. \tag{A.6}$$

The above optimality conditions imply the Euler equation (1)

$$\xi_t U_c(C_t) = \beta(1+i_t)\frac{P_t}{P_{t+1}}\xi_{t+1}U_c(C_{t+1}) \tag{A.7}$$

the pricing equation for long-term bonds

$$Q_t = \beta\frac{P_t}{P_{t+1}}\frac{\xi_{t+1}U_c(C_{t+1})}{\xi_t U_c(C_t)}(1+\delta Q_{t+1}) \tag{A.8}$$

and the implicit money demand function

$$\frac{v_m(M_t/P_t)}{U_C(C_t)} = \frac{i_t}{1+i_t} \tag{A.9}$$

from which it follows that the liquidity-preference function L in equation (5) is

$$L(Y, i_t) \equiv v_m^{-1}\left(U_C(Y)\frac{i_t}{1+i_t}\right), \tag{A.10}$$

where we used the equilibrium in the goods market $C_t = Y$.

A.2 Derivation of equation (10)

We use the following preference specification:

$$\sum_{t=t_0}^{\infty}\beta^{t-t_0}\xi_t\left[\ln C_t + \theta\ln\frac{M_t}{P_t}\right].$$

Consider equation (4),

$$\sum_{t=t_0+1}^{\infty}R_{t_0+1,t}\left(C_t + \frac{i_t}{1+i_t}\frac{M_t}{P_t}\right) = \frac{W_{t_0+1}}{P_{t_0+1}} + \sum_{t=t_0+1}^{\infty}R_{t_0+1,t}\left(Y_t - \frac{T_t}{P_t}\right).$$

and recall that in the long run $\xi_t = \bar{\xi}$ for each $t \geq t_0+1$. Note that, under this preference specification,

$$\frac{M_t}{P_t} = \theta C_t\frac{1+i_t}{i_t}$$

and moreover that $R_{t_0+1,t}C_t = \beta^{t-t_0-1}C_{t_0+1}$. We can then write (4) as

$$C_{t\,0+1} = \frac{1-\beta}{1+\theta}\left\{\frac{W_{t_0+1}}{P_{t_0+1}} + \sum_{t=t_0+1}^{\infty} R_{t_0+1,t}\left(Y_t - \frac{T_t}{P_t}\right)\right\}.$$

Use the assumption of constant endowment and real tax policy $T_t/P_t = \tau_t$, we can write it as

$$C_{t\,0+1} = \frac{1-\beta}{1+\theta}\left\{\frac{W_{t_0+1}}{P_{t_0+1}} + \sum_{t=t_0+1}^{\infty} R_{t_0+1,t}\left(Y - \tau\right)\right\}. \tag{A.11}$$

The above is the consumption demand, given income and policy, and for a given sequence of the real interest rate, captured by the discount factor $R_{t_0+1,t}$.

A.3 Nominal remittances rule and active interest-rate policy

In Section 4, we have determined the long-run price level using a real remittances policy. Here we show that it is possible, provided agents do not coordinate on a non-monetary equilibrium, to control the price level even by using a nominal remittances policy. In this case, we should change the interest rate rule to the following

$$1 + i_t = \frac{\Pi}{\beta}\left(\frac{P_t}{P_t^*}\right)^{\phi} \tag{A.12}$$

for each $t \geq t_0 + 1$ and for a positive parameter $\phi > 0$ in which

$$P_t^* = P^*\Pi^{t-t_0-1}.$$

for each $t \geq t_0 + 1$.

Let us consider the following remittances policy

$$T_t^C = \frac{i_t}{1+i_t}M_t + (1-\beta)\Pi^{t-t_0-1}N_{t_0}$$

which substituted into (12) implies

$$(1-\beta)\sum_{t=t_0+1}^{\infty}(\Pi\beta)^{t-t_0-1}\frac{1}{P_t} = \frac{1}{P_{t_0+1}}.$$

Moreover substituting (A.12) into (1), we obtain

$$P_{t+1} = \Pi \left(\frac{P_t}{P_t^*} \right)^{\phi} P_t$$

for each $t \geq t_0 + 1$. Comparing the above two equations, we can see that either $P_{t_0+1} = P^*$ and $P_{t+1} = \Pi P_t$ at each $t \geq t_0 + 1$, or prices are infinite at all times.

A.4 Solving the New-Keynesian model

In this section we provide details on the robustness analysis discussed in Section 5.

Consider the equations describing the private sector in the standard New-Keynesian model

$$\hat{Y}_t = \hat{Y}_{t+1} - \sigma \left[\hat{\imath}_t - r_t^n - (\pi_{t+1} - \pi) \right], \tag{A.13}$$

$$\pi_t - \pi = \beta(\pi_{t+1} - \pi) + \kappa \hat{Y}_t \tag{A.14}$$

where the variables are defined in the main text. The treasury follows a passive fiscal policy and is therefore irrelevant for price-level determination. Under the assumption that there are no long-term assets, key for the determination of prices is equation (19), which in a first-order approximation reads:

$$\hat{\imath}_{t-1} + \hat{n}_{t-1} - (\pi_t - \pi) = (1 - \beta)\hat{\tau}_t^C + \beta \hat{n}_t. \tag{A.15}$$

The two degrees of freedom implied by the system (A.13)–(A.15) are used to specify monetary policy along two dimensions: the path of the nominal interest rate and the path of remittances (or net worth), possibly as a function of other endogenous variables.

Now let $\tilde{n}_t \equiv \hat{\imath}_t + \hat{n}_t$, use it in equation (A.15), and iterate the latter forward, to get

$$\tilde{n}_{t-1} = \sum_{T=t}^{\infty} \beta^{T-t} \left[(1 - \beta)\hat{\tau}_T^C - \beta \hat{\imath}_T + (\pi_T - \pi) \right]. \tag{A.16}$$

From period $t_0 + 1$ onward, the central bank follows a policy that keeps real remittances and the nominal interest rate at their steady-state levels ($\hat{\tau}_t^C = \hat{\imath}_t = 0$), while no shocks to the natural rate occur: $r_t^n = 0$. Equation (A.16) therefore implies, for any $t \geq t_0 + 1$:

$$\tilde{n}_{t-1} = \sum_{T=t}^{\infty} \beta^{T-t}(\pi_T - \pi) = \pi_t - \pi + \beta \tilde{n}_t. \tag{A.17}$$

To get the stationary solution for any period $t \geq t_0 + 1$, we guess the policy functions

$$\tilde{n}_t = \lambda \tilde{n}_{t-1} \qquad \hat{Y}_t = \phi \tilde{n}_{t-1} \qquad \pi_t - \pi = \psi \tilde{n}_{t-1}$$

and use them in equations (A.13), (A.14) and (A.17) to retrieve the following system of equations

$$\phi = \lambda\phi + \psi\sigma\lambda \tag{A.18}$$

$$\psi = \psi\beta\lambda + \kappa\phi \tag{A.19}$$

$$1 = \psi + \beta\lambda \tag{A.20}$$

from which we can derive the equilibrium value of λ, ϕ and ψ given β, σ and κ. This completes the solution for any period $t \geq t_0 + 1$.

We move now to evaluate equilibrium at time t_0. In period t_0, a negative preference shock hits: $r^n_{t_0} = -r < 0$. The central bank responds cutting the nominal interest rate down to zero: $\hat{\imath}_{t_0} = -\bar{\imath}$. Moreover, consider the following equalities:

$$\tilde{n}_{t_0} = \hat{n}_{t_0} + \hat{\imath}_{t_0} = \log\left(\frac{N_{t_0}/P_{t_0}}{\bar{n}}\right) + \hat{\imath}_{t_0} = \hat{N}_{t_0} - p_{t_0} - \bar{\imath},$$

where $\hat{N}_{t_0} \equiv \log\left(N_{t_0}/\bar{n}\right)$ and $p_{t_0} \equiv \log(P_{t_0})$. Using the above in equations (A.13) and (A.14), evaluated at time t_0, and noting that $\pi_{t_0} = p_{t_0} - p_{t_0-1}$, we can write

$$\hat{Y}_{t_0} = \phi\tilde{n}_{t_0} + \sigma\psi\tilde{n}_{t_0} - \sigma(r - \bar{\imath}) = (\phi + \sigma\psi)\left(\hat{N}_{t_0} - p_{t_0} - \bar{\imath}\right) - \sigma(r - \bar{\imath})$$
$$p_{t_0} = p_{t_0-1} + \pi + \beta\psi\tilde{n}_{t_0} + \kappa\hat{Y}_{t_0} = p_{t_0-1} + \pi + \beta\psi\left(\hat{N}_{t_0} - p_{t_0} - \bar{\imath}\right) + \kappa\hat{Y}_{t_0},$$

from which simple algebra finally yields the short-run solution given by equations (22)–(23).